



6.4 A systematic approach to solving a system of linear equations





Introduction

- In this topic, we will
 - Introduce the next section
 - Give an overview of what will be covered





Systematic approach

- We have now seen that these two problems are equivalent:
 - Find a linear combination of n vectors in $\mathbf{R}^m$ that equals a target vector $\mathbf{v}$, also in $\mathbf{R}^m$

$$\alpha_1 \mathbf{u}_1 + \alpha_2 \mathbf{u}_2 + \alpha_3 \mathbf{u}_3 + \cdots + \alpha_n \mathbf{u}_n = \mathbf{v}$$

- Solve this system of m linear equations in n unknowns

$$a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 + \cdots + a_{1,n}x_n = b_1$$

$$a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 + \cdots + a_{2,n}x_n = b_2$$

$$a_{3,1}x_1 + a_{3,2}x_2 + a_{3,3}x_3 + \cdots + a_{3,n}x_n = b_3$$

$$\vdots$$

$$a_{m,1}x_1 + a_{m,2}x_2 + a_{m,3}x_3 + \cdots + a_{m,n}x_n = b_m$$





Systematic techniques

- Solving such algebraic equations by hand is difficult, at best, and tedious and error prone
 - We will introduce a technique that will solve such a system that eliminates the variable names
 - This technique can easily be programmed into a computer





Linear combinations to linear equations

- In the next few topics, we will:
 - Define matrices, its entries, column vectors
 - Define the augmented matrix equivalent to a given system of linear equations
 - Define the possible *row operations*
 - Define leading zeros in a row, the leading non-zero entry in a row, and row-echelon form
 - Describing numerically stable algorithms that manipulate the augmented matrix to find a solution to the system of linear equations:
 - Gaussian elimination with partial pivoting
 - Backward substitution





Summary

- Following this topic, you now
 - Understand the purpose of the next five topics
 - Know that at the end of this sequence of topics,
you will be able to either find a solution to a system of linear equations or determine no solution exists





References

- [1] https://en.wikipedia.org/wiki/System_of_linear_equations



Acknowledgments

None so far.



Colophon

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The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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